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Chapter 4
Some Types of HyperNeutrosophic Set (2): Complex, Single-Valued Triangular, Fermatean, and Linguistic Sets

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Abstract

This paper is a continuation of the work presented in [35]. The Neutrosophic Set provides a mathematical framework for managing uncertainty, characterized by three membership functions: truth, indeterminacy, and falsity. Recent advancements have introduced extensions such as the Hyperneutrosophic Set and SuperHyperneutrosophic Set to address more complex and multidimensional challenges.

In this study, we extend the Complex Neutrosophic Set, Single-Valued Triangular Neutrosophic Set, Fermatean Neutrosophic Set, and Linguistic Neutrosophic Set within the frameworks of Hyperneutrosophic Sets and SuperHyperneutrosophic Sets. Furthermore, we investigate their mathematical structures and analyze their connections with other set-theoretic concepts.

Keywords: Set Theory, SuperhyperNeutrosophic set, Neutrosophic Set, HyperNeutrosophic set

1 Preliminaries and Definitions

This section outlines the essential concepts and definitions required for the discussions in this paper. Basic set operations are utilized throughout this study. For a more comprehensive understanding of foundational set theory, readers are encouraged to consult references such as [24, 46, 48, 51]. Additionally, for fundamental operations and applications of Neutrosophic Sets, the relevant literature should be referred to as cited.

1.1 Neutrosophic, HyperNeutrosophic, and n-SuperHyperNeutrosophic Sets

To effectively address uncertainty and imprecision in decision-making, various set-theoretic frameworks have been developed. These include Fuzzy Sets [77–81], Intuitionistic Fuzzy Sets [7–12], Soft sets [45, 54, 55], and Neutrosophic Sets [26, 27, 34, 36–41, 43, 44, 64, 65, 69]. Additionally, advanced extensions such as Plithogenic Sets [25, 28–30, 42, 67, 68, 70] and Rough Sets [56–60] have been proposed. These models expand upon traditional set theories to better capture complex, multidimensional, and uncertain decision-making scenarios. Neutrosophic Sets extend Fuzzy Sets by introducing the concept of indeterminacy alongside truth and falsity [62–65]. This idea has been further developed into HyperNeutrosophic Sets and n-SuperHyperNeutrosophic Sets to handle even more complex scenarios [25, 31]. The following section provides their succinct definitions and relevant information.

Definition 1.1 (Neutrosophic Set)**.** [64, 65] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Definition 1.2 (HyperNeutrosophic Set)**.** (cf. [25, 31–33, 66]) Let X be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on X is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0,1]^3),$$

where $\mathcal{P}([0,1]^3)$ is the family of all non-empty subsets of the unit cube $[0,1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]^3$ is a set of neutrosophic membership triplets (T, I, F) that satisfy:

$$0 \le T + I + F \le 3.$$

Definition 1.3 (n-SuperHyperNeutrosophic Set)**.** (cf. [25, 31–33, 66]) Let X be a non-empty set. An *n-SuperHyperNeutrosophic Set (n-SHNS)* is a recursive generalization of Neutrosophic Sets and HyperNeutrosophic Sets. It is defined as a mapping:

$$\tilde{A}_n : \mathcal{P}_n(X) \to \mathcal{P}_n([0,1]^3),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$, the power set of X, and for $k \geq 2$,
$$\mathcal{P}_k(X) = \mathcal{P}(\mathcal{P}_{k-1}(X)),$$
representing the k-th nested family of non-empty subsets of X.
- $\mathcal{P}_n([0,1]^3)$ is defined similarly for the unit cube $[0,1]^3$.

For each $A \in \mathcal{P}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the following condition is satisfied:

$$0 \leq T + I + F \leq 3,$$

where T, I, F represent the degrees of truth, indeterminacy, and falsity for the n-th level subsets of X.

2 Results of This Paper

This section outlines the main results presented in this paper.

2.1 Complex Neutrosophic Set

A Complex Neutrosophic Set represents truth, indeterminacy, and falsity memberships as points on the complex unit circle [4–6, 13, 14, 53, 72]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

Definition 2.1 (Complex Neutrosophic Set)**.** [4, 5] Let U be a universe of discourse, and let $u \in U$ be an element. A *Complex Neutrosophic Set A* on U is characterized by three membership functions:

- *Truth-membership function* $T_A(u)$,
- *Indeterminacy-membership function* $I_A(u)$,
- *Falsity-membership function* $F_A(u)$,

where each function maps to the complex unit circle $\mathbb{C}_1$, i.e.,

$$T_A(u), I_A(u), F_A(u) \in \mathbb{C}_1 = \{z \in \mathbb{C} \mid |z| = 1\}.$$

The set A is represented as:

$$A = \{\langle u, T_A(u), I_A(u), F_A(u)\rangle \mid u \in U\}.$$

Definition 2.2 (Complex Hyperneutrosophic Set (CHNS))**.** Let X be a non-empty set, and let $\mathcal{P}(\mathcal{S})$ denote the family of all non-empty subsets of some region $\mathcal{S} \subseteq \mathbb{C}^3$. A *Complex Hyperneutrosophic Set (CHNS)* $\widetilde{C}$ on X is a mapping

$$\widetilde{C} : X \to \mathcal{P}(\mathcal{S}),$$

where each $\mathcal{S}$ is a subset of $\mathbb{C}^3$ that encodes the neutrosophic constraint in the complex domain. Concretely, for each $x \in X$,

$$\widetilde{C}(x) \subseteq \left\{(T, I, F) \in \mathcal{S} \subseteq \mathbb{C}^3 : |T| + |I| + |F| \leq 3\right\},$$

where (T, I, F) are the *complex truth, indeterminacy, and falsity values* for x. One might specifically choose $\mathcal{S} = \{(T, I, F) \in \mathbb{C}^3 : |T|, |I|, |F| \leq 1\}$ to mimic the real case.

Hence, each element $x \in X$ may have a *set* of possible complex neutrosophic membership triplets (T, I, F). Each such triplet satisfies:

$$|T| + |I| + |F| \leq 3.$$

Theorem 2.3. *Every Complex Neutrosophic Set is a special case of a Complex Hyperneutrosophic Set.*

Proof. A *Complex Neutrosophic Set (CNS)* A on X associates each $x \in X$ with exactly one triplet $(T_A(x), I_A(x), F_A(x)) \in \mathcal{S} \subseteq \mathbb{C}^3$, typically satisfying $|T_A(x)| + |I_A(x)| + |F_A(x)| \leq 3$. We embed this in Definition 2.2 by letting

$$\widetilde{C}(x) = \Big\{\big(T_A(x),\ I_A(x),\ F_A(x)\big)\Big\},$$

a singleton in $\mathcal{P}(\mathcal{S})$. The same magnitude constraint holds, so every CNS is realized as a single-valued (singleton) Complex Hyperneutrosophic Set. □

Theorem 2.4. *Every (real) Hyperneutrosophic Set is a special case of a Complex Hyperneutrosophic Set, by restricting complex values to real numbers.*

Proof. A standard *Hyperneutrosophic Set (HNS)* $\tilde{A}$ satisfies

$$\tilde{A}(x) \ \subseteq \ \Big\{(T, I, F) \in [0, 1]^3 : T + I + F \leq 3\Big\}.$$

In Definition 2.2, each membership is $\widetilde{C}(x) \subseteq \mathcal{S} \subseteq \mathbb{C}^3$. To match the real setting, force

$$T, I, F \ \in \ [0, 1] \subset \mathbb{C}, \quad \text{(imaginary part = 0)},$$

and impose $|T| + |I| + |F| = T + I + F \leq 3$. Then

$$\widetilde{C}(x) = \Big\{(T, I, F) \in [0, 1]^3 : T + I + F \leq 3\Big\}.$$

Hence we recover the real HNS as a restriction of CHNS to purely real triplets. Therefore, every real HNS is embedded in a CHNS by ignoring any imaginary component. □

Definition 2.5 (Complex n-SuperHyperneutrosophic Set (C-n-SHNS))**.** Let X be a non-empty set, and define

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad \text{for } k \geq 2.$$

Similarly, let $\mathcal{P}_n(\mathcal{S})$ denote the n-th nested power set of a region $\mathcal{S} \subseteq \mathbb{C}^3$. A *Complex n-SuperHyperneutrosophic Set (C-n-SHNS)* is a mapping

$$\widetilde{C}_n : \mathcal{P}_n(X) \ \longrightarrow \ \mathcal{P}_n(\mathcal{S}),$$

such that for any $A \in \mathcal{P}_n(X)$ and any triplet $(T, I, F) \in \widetilde{C}_n(A) \subseteq \mathcal{S} \subseteq \mathbb{C}^3$, we have

$$|T| + |I| + |F| \ \leq \ 3.$$

Thus, each n-th level subset $A \subseteq X$ is assigned a *set* of complex-valued membership triplets $\big(T, I, F\big)$ satisfying the magnitude-sum condition ≤ 3, just as in the single-level CHNS, but now extended to n-nested subsets of X.

Theorem 2.6. *Every Complex Hyperneutrosophic Set is a special case of a Complex n-SuperHyperneutrosophic Set (C-n-SHNS), obtained by setting $n = 1$.*

Proof. A *Complex Hyperneutrosophic Set (CHNS)* $\widetilde{C}$ has $\widetilde{C}(x) \subseteq \mathcal{S} \subseteq \mathbb{C}^3$. In Definition 2.5, let $n = 1$. Then

$$\widetilde{C}_1 : \mathcal{P}_1(X) = \mathcal{P}(X) \ \longrightarrow \ \mathcal{P}_1(\mathcal{S}) = \mathcal{P}(\mathcal{S}).$$

We can define:

$$\widetilde{C}_1(\{x\}) := \widetilde{C}(x), \quad \text{and possibly set } \widetilde{C}_1(A) = \varnothing \text{ for } A \neq \{x\}.$$

Hence, restricting to singletons $\{x\} \subseteq X$ reproduces precisely the membership sets $\widetilde{C}(x)$. The constraint $|T| + |I| + |F| \leq 3$ persists. Therefore, any CHNS is embedded in a C-1-SHNS as a single-level scenario. □

Theorem 2.7. *Every (real) n-SuperHyperneutrosophic Set is a special case of a Complex n-SuperHyperneutrosophic Set by restricting triplets to real values.*

Proof. An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ satisfies

$$\tilde{A}_n(A) \subseteq \left\{(T, I, F) \in [0,1]^3 : T + I + F \leq 3\right\}$$

for each $A \in \mathcal{P}_n(X)$. Compare with Definition 2.5, where

$$\widetilde{C}_n(A) \subseteq \mathcal{S} \subseteq \mathbb{C}^3 \quad \text{with} \quad |T| + |I| + |F| \leq 3.$$

Restrict each T, I, F to be real and non-negative, i.e. let

$$T, I, F \in [0,1] \subset \mathbb{C} \quad \text{(imaginary part = 0)},$$

and impose $T + I + F \leq 3$. Define

$$\widetilde{C}_n(A) = \left\{(T, I, F) \in [0,1]^3 : T + I + F \leq 3,\ (T, I, F) \in \tilde{A}_n(A)\right\}.$$

Hence, the real n-SuperHyperneutrosophic membership sets appear as a restriction of the complex domain. Therefore, every n-SHNS is included in a C-n-SHNS by limiting the membership triplets to real values. □

2.2 Single-valued triangular neutrosophic set

A Single-Valued Triangular Neutrosophic Set uses triangular fuzzy numbers to represent truth, indeterminacy, and falsity membership functions for elements in a set [1, 2, 20, 22, 23, 47, 49, 71]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

Definition 2.8 (Single-Valued Triangular Neutrosophic Set)**.** (cf. [2,23]) Let X be a universe of discourse. A *Single-Valued Triangular Neutrosophic Set (SVTNS) A* in X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\},$$

where each of $T_A(x), I_A(x), F_A(x)$ is a *triangular fuzzy number*, expressed as:

$$T_A(x) = \langle T_l, T_m, T_r\rangle, \quad I_A(x) = \langle I_l, I_m, I_r\rangle, \quad F_A(x) = \langle F_l, F_m, F_r\rangle.$$

The real numbers (T_l, T_m, T_r), (I_l, I_m, I_r), and (F_l, F_m, F_r) satisfy $T_l \leq T_m \leq T_r$, $I_l \leq I_m \leq I_r$, $F_l \leq F_m \leq F_r$. Each triangular fuzzy membership is defined piecewise, e.g.:

$$T_A(x) = \begin{cases} \frac{x - T_l}{T_m - T_l}, & \text{if } T_l \leq x \leq T_m, \\ \frac{T_r - x}{T_r - T_m}, & \text{if } T_m \leq x \leq T_r, \\ 0, & \text{otherwise.} \end{cases}$$

Similar piecewise definitions hold for $I_A(x)$ and $F_A(x)$.

Intuitively,

- $T_A(x)$ represents the "truth-membership" of x with a triangular shape,
- $I_A(x)$ represents the "indeterminacy-membership" of x,
- $F_A(x)$ represents the "falsity-membership" of x.

This structure extends classical Single-Valued Neutrosophic Sets by allowing each membership function to follow a triangular distribution over X.

Definition 2.9 (Single-Valued Triangular Hyperneutrosophic Set (SVTHNS))**.** Let X be a non-empty set, and let

$$\mathcal{T} = \left\{\langle t_l, t_m, t_r\rangle : t_l \leq t_m \leq t_r, \quad t_l, t_m, t_r \in \mathbb{R}\right\}$$

be the set of all possible triangular fuzzy numbers on the real line. A *Single-Valued Triangular Hyperneutrosophic Set (SVTHNS) $\widetilde{S}$* on X is a mapping

$$\widetilde{S} : X \longrightarrow \mathcal{P}(\mathcal{T}^3),$$

where for each $x \in X$,

$$\widetilde{S}(x) \subseteq \Big\{(T, I, F) \in \mathcal{T}^3 \;\Big|\; \text{each of } T, I, F \text{ is a triangular fuzzy number, and } \max(T_r, I_r, F_r) \leq M\Big\},$$

for some real upper bound $M > 0$ (analogous to constraints like ≤ 3 in standard neutrosophic sets, though one might specify additional conditions if desired).

In simpler terms, each $x \in X$ can be assigned a *set* of triplets (T, I, F), each of which is made of triangular fuzzy numbers for truth, indeterminacy, and falsity, respectively.

Theorem 2.10. *Every Single-Valued Triangular Neutrosophic Set is a special case of a Single-Valued Triangular Hyperneutrosophic Set.*

Proof. A *Single-Valued Triangular Neutrosophic Set (SVTNS) A* on X associates each $x \in X$ with exactly one triplet of triangular fuzzy numbers $(T_A(x), I_A(x), F_A(x))$. To embed this into Definition 2.9, define

$$\widetilde{S}(x) = \Big\{(T_A(x), I_A(x), F_A(x))\Big\},$$

i.e. a singleton set. Each $(T_A(x), I_A(x), F_A(x)) \in \mathcal{T}^3$ is already guaranteed by the triangular membership definitions for truth, indeterminacy, and falsity. Thus, every SVTNS is reproduced as a Single-Valued Triangular Hyperneutrosophic Set with singletons. □

Theorem 2.11. *Every (real) Hyperneutrosophic Set is a special case of a Single-Valued Triangular Hyperneutrosophic Set, by restricting the triangular fuzzy numbers to degenerate points.*

Proof. A *Hyperneutrosophic Set (HNS)* $\tilde{A}$ maps each $x \in X$ to a (possibly multi-valued) subset of $[0, 1]^3$, e.g. triplets $(T, I, F) \in [0, 1]^3$. In Definition 2.9, each membership is a subset of $\mathcal{T}^3$. To mimic the real HNS scenario, let each triangular fuzzy number degenerate to a single real point, i.e.

$$\langle t_l, t_m, t_r\rangle = \langle r, r, r\rangle, \quad r \in [0, 1].$$

Then each $(T, I, F) \in \mathcal{T}^3$ effectively behaves like a real triple in $[0, 1]^3$. Define

$$\widetilde{S}(x) = \Big\{(\langle T, T, T\rangle, \langle I, I, I\rangle, \langle F, F, F\rangle) \;\Big|\; (T, I, F) \in \tilde{A}(x)\Big\}.$$

Hence, if $\tilde{A}(x) \subseteq [0, 1]^3$, we have a corresponding set of triangular triplets degenerating to points. Thus, every real-based HNS can be viewed as a degenerate special case of an SVTHNS. □

Definition 2.12 (Single-Valued Triangular n-SuperHyperneutrosophic Set (SVT-n-SHNS))**.** Let X be a non-empty set, and define recursively:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad \text{for } k \geq 2.$$

Likewise, let $\mathcal{T}$ be the set of all triangular fuzzy numbers on the real line, and consider $\mathcal{P}_n(\mathcal{T}^3)$ for n-nested subsets of $\mathcal{T}^3$.

A *Single-Valued Triangular n-SuperHyperneutrosophic Set (SVT-n-SHNS)* is a mapping

$$\widetilde{S}_n : \mathcal{P}_n(X) \longrightarrow \mathcal{P}_n(\mathcal{T}^3),$$

such that for each $A \in \mathcal{P}_n(X)$, $\widetilde{S}_n(A) \subseteq \mathcal{T}^3$ (or a set of such triplets), each triplet being $\langle T, I, F\rangle \in \mathcal{T}^3$ in the triangular fuzzy sense.

Theorem 2.13. *Every Single-Valued Triangular Hyperneutrosophic Set is a special case of a Single-Valued Triangular n-SuperHyperneutrosophic Set (SVT-n-SHNS), obtained by letting* $n = 1$.

Proof. A *Single-Valued Triangular Hyperneutrosophic Set (SVTHNS)* $\widetilde{S}$ has $\widetilde{S}(x) \subseteq \mathcal{T}^3$. In Definition 2.12, let $n = 1$. Then

$$\widetilde{S}_1 : \mathcal{P}_1(X) = \mathcal{P}(X) \;\to\; \mathcal{P}_1(\mathcal{T}^3) = \mathcal{P}(\mathcal{T}^3).$$

We can define

$$\widetilde{S}_1(\{x\}) := \widetilde{S}(x), \quad \widetilde{S}_1(A) = \varnothing \text{ for } A \neq \{x\}.$$

Hence, restricting to singletons $\{x\} \subseteq X$ recovers precisely the membership sets from $\widetilde{S}(x)$. Therefore, every SVTHNS is subsumed under an SVT-1-SHNS scenario. □

Theorem 2.14. *Every n-SuperHyperneutrosophic Set is a special case of a Single-Valued Triangular n-SuperHyperneutrosophic Set, by making each triangular membership degenerate to a single real value.*

Proof. An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ satisfies

$$\tilde{A}_n(A) \;\subseteq\; [0,1]^3,$$

or some similar real domain for each $A \in \mathcal{P}_n(X)$. Compare with Definition 2.12, where $\widetilde{S}_n(A) \subseteq \mathcal{T}^3$. If we let each triangular fuzzy number $\langle \alpha_l, \alpha_m, \alpha_r \rangle$ collapse to $\langle r, r, r \rangle \in [0,1]$ for some real $r \in [0,1]$, then

$$\langle T_l, T_m, T_r \rangle = \langle T, T, T \rangle, \quad \langle I_l, I_m, I_r \rangle = \langle I, I, I \rangle, \quad \langle F_l, F_m, F_r \rangle = \langle F, F, F \rangle.$$

Thus, define

$$\widetilde{S}_n(A) = \Big\{ \big(\langle T,T,T \rangle, \langle I,I,I \rangle, \langle F,F,F \rangle \big) \;\Big|\; (T,I,F) \in \tilde{A}_n(A) \Big\}.$$

This exactly reproduces the real triplet approach in the n-SuperHyperneutrosophic Set. Hence, n-SHNS emerges as a degenerate special case of SVT-n-SHNS. □

2.3 Fermatean Neutrosophic Set

A Fermatean Neutrosophic Set represents truth, indeterminacy, and falsity memberships satisfying $(T_F)^3 + (I_F)^3 + (F_F)^3 \leq 2$ [3, 15–17, 21, 50, 61]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

Definition 2.15 (Fermatean Neutrosophic Set)**.** (cf. [16, 50, 61]) A *Fermatean Neutrosophic Set (FNS)* on a universe A is defined as:

$$F = \{ \langle x, T_F(x), I_F(x), F_F(x) \rangle \mid x \in A \},$$

where the following conditions hold:

1. $T_F(x), I_F(x), F_F(x) \in [0,1]$ for all $x \in A$,
2. $(T_F(x))^3 + (I_F(x))^3 + (F_F(x))^3 \leq 2.$

Here:

- $T_F(x)$: The degree of truth of the element x to the set F,
- $I_F(x)$: The degree of indeterminacy of x to the set F,
- $F_F(x)$: The degree of falsity of x to the set F.

Definition 2.16 (Fermatean Hyperneutrosophic Set (FHNS))**.** Let X be a non-empty set, and let

$$\mathcal{F} \;\subseteq\; \{ (T,I,F) \in [0,1]^3 : T^3 + I^3 + F^3 \;\leq\; 2 \}.$$

A *Fermatean Hyperneutrosophic Set (FHNS)* $\widetilde{F}$ on X is a mapping

$$\widetilde{F} : X \;\to\; \mathcal{P}(\mathcal{F}),$$

where for each $x \in X$,

$$\widetilde{F}(x) \;\subseteq\; \big\{ (T,I,F) \in [0,1]^3 : T^3 + I^3 + F^3 \;\leq\; 2 \big\}.$$

In simpler terms, each $x \in X$ is assigned a *set* of Fermatean triplets (T, I, F), each triplet satisfying

$$T^3 + I^3 + F^3 \;\leq\; 2, \quad (T,I,F) \in [0,1]^3.$$

Theorem 2.17. *Every Fermatean Neutrosophic Set is a special case of a Fermatean Hyperneutrosophic Set.*

Proof. A *Fermatean Neutrosophic Set (FNS) F* on A assigns each $x \in A$ a single triplet $\big(T_F(x), I_F(x), F_F(x)\big) \in [0,1]^3$ satisfying $\big(T_F(x)\big)^3 + \big(I_F(x)\big)^3 + \big(F_F(x)\big)^3 \le 2$. By Definition 2.16, we allow each x a *set* of such triplets. Let

$$\widetilde{F}(x) = \Big\{\big(T_F(x), I_F(x), F_F(x)\big)\Big\},$$

a singleton set. Then the same cubic constraint holds, so each single-valued FNS is embedded in the FHNS framework as a degenerate (singleton) case. □

Theorem 2.18. *Every (standard) Hyperneutrosophic Set is a special case of a Fermatean Hyperneutrosophic Set when the cubic constraint is omitted or replaced by the usual sum constraint.*

Proof. A standard *Hyperneutrosophic Set (HNS) $\tilde{A}$* has

$$\tilde{A}(x) \;\subseteq\; [0,1]^3 \quad \text{with sum constraints e.g. } T + I + F \le 3.$$

In Definition 2.16, we require $T^3 + I^3 + F^3 \le 2$. Observe that if we *ignore* or *do not enforce* the Fermatean condition, or equivalently treat the entire region $[0,1]^3$ as permissible, we obtain a typical HNS definition. Formally, define

$$\widetilde{F}(x) = \tilde{A}(x) \quad \text{and treat the Fermatean constraint as optional.}$$

Hence the real hyperneutrosophic membership sets appear as a sub-case within FHNS by ignoring the cubic bound. Therefore, a standard HNS emerges as a special scenario of FHNS lacking the Fermatean constraint. □

Definition 2.19 (Fermatean n-SuperHyperneutrosophic Set (F-n-SHNS))**.** Let X be a non-empty set. Define recursively:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad \text{for } k \ge 2.$$

Let

$$\mathcal{F} \;=\; \big\{(T, I, F) \in [0,1]^3 : T^3 + I^3 + F^3 \le 2\big\}.$$

Then the *Fermatean n-SuperHyperneutrosophic Set (F-n-SHNS)* is a mapping

$$\widetilde{F}_n : \mathcal{P}_n(X) \;\longrightarrow\; \mathcal{P}_n(\mathcal{F}),$$

meaning that for any $A \in \mathcal{P}_n(X)$,

$$\widetilde{F}_n(A) \;\subseteq\; \mathcal{F} \;=\; \big\{(T, I, F) \in [0,1]^3 :\, T^3 + I^3 + F^3 \le 2\big\}.$$

Thus, each n-th level subset $A \subseteq X$ is mapped to a *set* of Fermatean membership triplets, all satisfying $T^3 + I^3 + F^3 \le 2$.

Theorem 2.20. *Every Fermatean Hyperneutrosophic Set is a special case of a Fermatean n-SuperHyperneutrosophic Set (F-n-SHNS), by setting $n = 1$.*

Proof. A *Fermatean Hyperneutrosophic Set (FHNS) $\widetilde{F}$* has $\widetilde{F}(x) \subseteq \{(T, I, F) \in [0,1]^3 : T^3 + I^3 + F^3 \le 2\}$. In Definition 2.19, let $n = 1$, so $\mathcal{P}_1(X) = \mathcal{P}(X)$. Define

$$\widetilde{F}_1(\{x\}) := \widetilde{F}(x), \quad \widetilde{F}_1(A) = \varnothing \text{ for } A \neq \{x\}.$$

Hence each singleton $\{x\} \subseteq X$ recovers precisely the membership sets from the original FHNS. The constraint $T^3 + I^3 + F^3 \le 2$ remains identical. Consequently, we embed FHNS as a special single-level version of F-n-SHNS. □

Theorem 2.21. *Every (standard) n-SuperHyperneutrosophic Set is a special case of a Fermatean n-SuperHyperneutrosophic Set by ignoring the Fermatean cubic constraint.*

Proof. An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ satisfies

$$\tilde{A}_n(A) \subseteq [0,1]^3$$

for each $A \in \mathcal{P}_n(X)$. In Definition 2.19, we require $(T, I, F) \in [0,1]^3$ with $T^3 + I^3 + F^3 \leq 2$. If we do not impose that extra condition (or treat it as automatically satisfied for all $(T, I, F) \in [0,1]^3$), we revert to an n-SHNS. Formally, define

$$\widetilde{F_n}(A) = \tilde{A}_n(A),$$

and skip the cubic constraint. Thus, the usual n-SuperHyperneutrosophic membership sets appear as a subcase of the Fermatean approach. Hence, every n-SHNS is included in F-n-SHNS by discarding the Fermatean requirement. □

2.4 Linguistic HyperNeutrosophic Set

A Linguistic Neutrosophic Set integrates linguistic terms with truth, indeterminacy, and falsity degrees, addressing uncertainty and vagueness in linguistic assessments [18, 19, 52, 73–76, 82]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

Definition 2.22. (cf. [18,52,75]) Let X be a non-empty set (universe of discourse) and $S = \{s_i \mid i = 1, 2, \ldots, t\}$ a linguistic term set, where each s_i represents a linguistic term (e.g., "low", "medium", "high"). A *Linguistic Neutrosophic Set (LNS)* is defined as:

$$\mathcal{S} = \{\langle s_x, T(s_x), I(s_x), F(s_x)\rangle \mid s_x \in S\},$$

where $T(s_x), I(s_x), F(s_x)$ are linguistic truth-membership, indeterminacy-membership, and falsity-membership values, respectively, for the term s_x. These are linguistic variables representing the degrees of truth, indeterminacy, and falsity associated with s_x.

Each $T(s_x), I(s_x), F(s_x)$ satisfies the following properties:

$$T(s_x), I(s_x), F(s_x) \in [0,1], \quad \forall s_x \in S.$$

Furthermore, the operational laws for linguistic neutrosophic numbers (LNNs) $l_1 = \langle T_1, I_1, F_1\rangle$ and $l_2 = \langle T_2, I_2, F_2\rangle$ are defined as:

$$\begin{aligned}
l_1 \oplus l_2 &= \langle T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2\rangle, \\
l_1 \ominus l_2 &= \langle \frac{T_1 - T_2}{1 - T_2}, \frac{I_1}{I_2}, \frac{F_1}{F_2}\rangle, \\
l_1 \otimes l_2 &= \langle T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2\rangle, \\
l_1 \oslash l_2 &= \langle \frac{T_1}{T_2}, \frac{I_1 - I_2}{1 - I_2}, \frac{F_1 - F_2}{1 - F_2}\rangle, \\
\lambda l_1 &= \langle 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda\rangle, \\
l_1^\lambda &= \langle T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda\rangle.
\end{aligned}$$

Definition 2.23 (Linguistic Hyperneutrosophic Set (LHNS))**.** Let $S = \{s_1, s_2, \ldots, s_t\}$ be a linguistic term set, and let $\mathcal{L} \subseteq [0,1]^3$ represent the domain of possible *linguistic neutrosophic numbers* (LNNs), subject to

$$T(s_x),\ I(s_x),\ F(s_x) \in [0,1] \quad \text{for each } s_x \in S.$$

A *Linguistic Hyperneutrosophic Set (LHNS)* is a mapping

$$\widetilde{L} : S \ \to\ \mathcal{P}(\mathcal{L}),$$

where each $\widetilde{L}(s_x) \subseteq \mathcal{L}$. Concretely, for each linguistic term $s_x \in S$,

$$\widetilde{L}(s_x) \ \subseteq\ \big\{(T, I, F) \in [0,1]^3 :\ \text{each } (T, I, F) \text{ can be interpreted as a linguistic neutrosophic number}\big\}.$$

Hence, each linguistic term s_x is assigned a *set* of LNN-based triplets (T, I, F), capturing diverse or uncertain opinions regarding truth, indeterminacy, and falsity degrees.

Theorem 2.24. *Every Linguistic Neutrosophic Set (LNS) is a special case of a Linguistic Hyperneutrosophic Set (LHNS).*

Proof. A *Linguistic Neutrosophic Set (LNS)* $\mathcal{S}$ assigns exactly one linguistic neutrosophic number $(T(s_x), I(s_x), F(s_x))$ to each $s_x \in S$. In Definition 2.23, we allow each s_x a *set* of such triplets. Let

$$\widetilde{L}(s_x) = \Big\{\big(T(s_x), I(s_x), F(s_x)\big)\Big\},$$

i.e., a singleton set. Hence the single-valued LNS membership is realized in the LHNS context as a singleton for each linguistic term. Consequently, every LNS is a degenerate (singleton) LHNS. □

Theorem 2.25. *Every standard Hyperneutrosophic Set is a special case of a Linguistic Hyperneutrosophic Set by removing the linguistic layer and focusing on numeric real values in* $[0, 1]$.

Proof. A standard *Hyperneutrosophic Set (HNS)* $\tilde{A}$ is typically a mapping from a non-empty set X (or S) to subsets of $[0, 1]^3$ with constraints on (T, I, F). In Definition 2.23, we define each membership set in $\mathcal{L} \subseteq [0, 1]^3$ with a linguistic interpretation. If we ignore or do not require the linguistic semantics and treat (T, I, F) purely as real-based memberships (i.e., ignoring the LNN operations or labels), we recover the same structure as HNS. Formally, define

$$\widetilde{L}(s_x) = \tilde{A}(s_x),$$

and let the linguistic viewpoint be optional. This reproduces the real hyperneutrosophic membership sets. Thus, ignoring the linguistic dimension reverts LHNS to a standard HNS. □

Definition 2.26 (Linguistic n-SuperHyperneutrosophic Set (L-n-SHNS))**.** Let $S = \{s_1, s_2, \dots, s_t\}$ be a linguistic term set, and let $\mathcal{L} \subseteq [0, 1]^3$ denote the domain of possible LNN-based triplets (T, I, F). Define:

$$\mathcal{P}_1(S) = \mathcal{P}(S), \quad \mathcal{P}_k(S) = \mathcal{P}\big(\mathcal{P}_{k-1}(S)\big) \quad \text{for } k \geq 2.$$

Likewise, consider $\mathcal{P}_n(\mathcal{L})$, the n-th nested family of subsets of $\mathcal{L}$.

A *Linguistic n-SuperHyperneutrosophic Set (L-n-SHNS)* is a mapping

$$\widetilde{L}_n : \mathcal{P}_n(S) \longrightarrow \mathcal{P}_n(\mathcal{L}),$$

meaning that for each $A \in \mathcal{P}_n(S)$, $\widetilde{L}_n(A) \subseteq \mathcal{L}$ is a *set* of LNN triplets in $[0, 1]^3$, capturing the linguistic truth, indeterminacy, and falsity degrees for the n-th level subset A.

Theorem 2.27. *Every Linguistic Hyperneutrosophic Set is a special case of a Linguistic n-SuperHyperneutrosophic Set (L-n-SHNS) for n* $= 1$.

Proof. A *Linguistic Hyperneutrosophic Set (LHNS)* $\widetilde{L}$ has $\widetilde{L}(s_x) \subseteq \mathcal{L} \subseteq [0, 1]^3$ for each $s_x \in S$. In Definition 2.26, let $n = 1$. Then

$$\widetilde{L}_1 : \mathcal{P}_1(S) = \mathcal{P}(S) \longrightarrow \mathcal{P}_1(\mathcal{L}) = \mathcal{P}(\mathcal{L}).$$

For each singleton $\{s_x\} \subseteq S$, define

$$\widetilde{L}_1(\{s_x\}) := \widetilde{L}(s_x), \quad \widetilde{L}_1(A) = \varnothing \text{ for } A \neq \{s_x\}.$$

Hence, restricting to singletons recovers the LHNS membership sets. Thus, any LHNS is embedded in an L-1-SHNS as a single-level scenario. □

Theorem 2.28. *Every standard n-SuperHyperneutrosophic Set is a special case of a Linguistic n-SuperHyperneutrosophic Set, by disregarding the linguistic interpretation and using real values in* $[0, 1]^3$.

Proof. An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ is a mapping from $\mathcal{P}_n(U)$ (for some universe U) to subsets of $[0, 1]^3$. In Definition 2.26, we map $\mathcal{P}_n(S)$ to subsets of $\mathcal{L} \subseteq [0, 1]^3$, with a linguistic dimension. If we ignore the linguistic labeling (or treat s_x as just an element in U) and interpret (T, I, F) purely as real degrees in $[0, 1]^3$, the structure coincides with a standard n-SHNS. Formally:

$$\widetilde{L}_n(A) = \tilde{A}_n(A), \quad \text{and ignore linguistic semantics.}$$

Hence, each membership set in $[0, 1]^3$ appears identically in the L-n-SHNS. Therefore, an n-SHNS emerges as a sub-case of L-n-SHNS by skipping the linguistic layer. □

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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